INFLUENCE OF PRESSURE DROP ON FLOW OF LOOSE MATERIAL THROUGH VERTICAL CHANNELS

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The analogy between flow of a fluid and flow of a loose material out from the space beneath a vault is used, in conjunction with an investigation of loose-material flow, as a basis for a proposed equation for the weight flow rate of loose material in the presence of a pressure differential.

In many types of equipment, we find loose materials flowing out through a hole in the horizontal bottom of a vertical channel; this can occur in the presence of a pressure differential between the top and bottom of the layer of material in the channel. We shall consider the influence of the gas flow produced by the pressure differential on the flow of loose materials through a hole in the horizontal body; we proceed from the following assumptions.

It has been shown [1] that the rate of gravitational outflow is determined by the pressure exerted by the material on the plane of the discharge hole. The dynamic vault that forms in gravitational flow near a discharge hole tends to relieve the plane from the pressure exerted by the higher layers of material [2]. Owing to the existence of the dynamic vault, the pressure at the plane of the hole equals the weight of the particles filling the space beneath the vault [3, 4].

It is natural to assume that such a vault, whose dimensions and form depend on the diameter of the hole and the mechanical properties of the material [5], also exists when a gas flows through a dense layer of material in the channel. In such case, the plane of the discharge hole will experience the pressure exerted by the volume of loose material under the vault, and the additional pressure determined by the difference of pressures in the gas flow between the top and bottom of the space under the vault. This conclusion has been confirmed elsewhere [6, 7]; it was shown that the rate of outflow is not affected by the entire pressure difference across the layer, but only by that portion equaling the difference in pressures "near the hole."

Thus we can make use of the familiar fluid-flow equation

$$G=\mu\gamma(1-\varepsilon_{\mathrm{v}})F\sqrt{2g\frac{\gamma(1-\varepsilon_{\mathrm{v}})h_{\mathrm{v}}+\Delta P_{\mathrm{v}}}{\gamma(1-\varepsilon_{\mathrm{v}})}}, \tag{1}$$

which can be written as follows:

$$G=\mu\left(\frac{1-\varepsilon_{\mathrm{v}}}{1-\varepsilon_{\mathrm{st}}}\right)^{0,5}\gamma_{\mathrm{b}}F\sqrt{2g\frac{\gamma_{\mathrm{b}}\left(\frac{1-\varepsilon_{\mathrm{v}}}{1-\varepsilon_{\mathrm{st}}}\right)h_{\mathrm{st}}+\Delta P_{\mathrm{st}}}{\gamma_{\mathrm{b}}}}. \tag{2}$$

Within the zone of the material that is far distant from the discharge hole, the pressure gradient is constant over the layer height since the gas velocity and the characteristics of the moving layer are also constant. We assume in approximation that the pressure gradient remains constant up to the level that corresponds to the height of the dynamic vault, below which the gradient increases by a factor of k, owing to the reduction in the flow cross section (when $D_{\mathrm{VC}} > D$); thus we obtain

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TABLE 1. Comparison of Predicted Values of Discharge Coefficient

D/d		7	10	20	30	40	50	60
μ_p	Eq. (9)	0,23	0,285	0,39	0,445	0,48	0,50	0,52
	[6]	0,285	0,33	0,39	0,41	0,42	0,43	0,435
Error, %		—19,3	—13,7	0	+8,5	+14,3	+16,2	+19,5

TABLE 2. Characteristics of Loose Materials

Material	d, mm	γ_b, kg/m³
River sand	0,22	1370
"	0,33	1370
"	0,8	1450
Slag pellets	1,4	1680
Millet	2,0	850
Corundum	0,33	1920

$$\Delta P_v = \frac{\Delta P}{\frac{H-h_v}{k}+h_v}\, h_v. \tag{3}$$

It has been shown in [1, 3-5] that the pressure exerted by the loose material on the plane of the discharge hole, which is nearly the same as the product of the bulk density of the material by the diameter of the hole, can be taken to equal

$$h_v\,\frac{1-\varepsilon_v}{1-\varepsilon_{st}} = D. \tag{4}$$

From (3), (4), and (2), we have

$$G=\mu\left(\frac{1-\varepsilon_v}{1-\varepsilon_{st}}\right)^{0.5}\gamma_b F\sqrt{2gD\left\{1+\frac{\Delta P}{\gamma_b\left[\frac{H-D\left(\frac{1-\varepsilon_{st}}{1-\varepsilon_v}\right)}{k\left(\frac{1-\varepsilon_{st}}{1-\varepsilon_v}\right)}+D\right]}\right\}} \tag{5}$$

The published literature contains contradictory opinions as to the value of the average porosity of the space under the vault: it is ordinarily assumed that $\varepsilon_v > \varepsilon_{st}$ [4, 5], but it was asserted in [8] that the lowest porosity is found near the discharge hole, i.e., in the space under the vault. It can be assumed that ε_v depends on the size of the hole, the physical properties of the material, etc., i.e., fundamentally on those parameters that must determine the values of the coefficients μ and k. It is therefore desirable to let $\mu_p = \mu(1-\varepsilon_v)/(1-\varepsilon_{st})^{0.5}$ and $k_p = k(1-\varepsilon_{st})/(1-\varepsilon_v)$. Using μ_p and k_p, and remembering that the height of the layer is noticeably greater than the diameter of the hole, on the basis of (5) we can write

$$G=\mu_p\gamma_b F\sqrt{2gD\left[1+\frac{\Delta P}{\gamma_b\left(\frac{H-D}{k_p}+D\right)}\right]}. \tag{6}$$

Proceeding with the analogy between fluid flow and the flow of loose material from a vault volume, we assume that the discharge coefficient in (6) does not depend on the weight flow rate. In this case, we can assume that it is identical for both gravitational flow and for gas filtration, provided that the flow takes place in the same medium (in the atmosphere, for example).

Comparing the relationship

$$G_{gr}=\frac{2.15}{1+11.8\,(d/D)}\,\gamma_b D^{2.5} \tag{7}$$

posed in [9] and the equation

$$G_{gr}=\mu_p\gamma_b F\sqrt{2gD}, \tag{8}$$

which can be obtained from Eq. (6) if we let $\Delta P = 0$, we find

$$\mu_p=\frac{1}{1.61+19\,(d/D)}. \tag{9}$$

Table 1 compares the values of discharge coefficient predicted by Eq. (9) and by the equation obtained from the relationship proposed in [6] for the weight flow rate of loose material flowing in the atmosphere through a hole in a horizontal bottom and flowing in the presence of excess pressure above the layer of

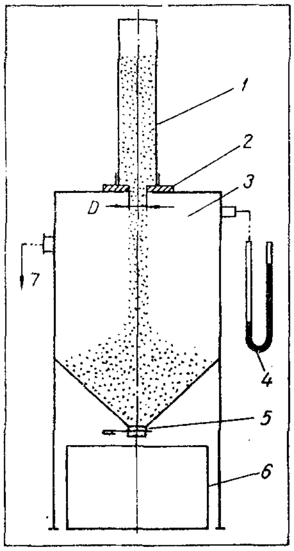

Fig. 1. Experimental setup: 1) vertical channel; 2) replaceable diaphragm; 3) vessel; 4) differential manometer; 5) hole; 6) collector; 7) to blower.

material in the channel. The research of [6] was carried out at D/D < 60, in the main.

Table 1 supports the above assumption that μ_p is independent of the weight flow rate and shows that we can take the discharge coefficients occurring in (6) and (8) to be identical (to within ±19.5%). Thus from (6) and (8) we have

$$\frac{G}{G_{gr}} = \sqrt{1 + \frac{\Delta P}{\gamma_b \left(\frac{H - D}{k_p} + D\right)}}. \tag{10}$$

The materials whose characteristics are given in Table 2 were investigated with air filtering through the layer of material in the channel.

It should be noted that in talking about loose materials, we are referring to ideally free-flowing media, i.e., adhesive forces between particles are absent or slight.

The experimental setup (Fig. 1) consisted of the vessel 3, in which excess pressure for vacuum was created, a vertical channel 1, the interchangeable diaphragm 2 with the discharge hole, and the collector 6.

The pressure in the vessel under the discharge hole was measured by a type MMN micromanometer, or a U-tube differential manometer 4. Glass tubes of 29, 50, and 77 mm inside diameter were used as the vertical channel. The diameter of the discharge hole was varied from 6.7 to 40 mm, the pressure drop across the layer from −4000 to +4000 N/m^2, and the height of the layer in the vertical channel from 100 to 500 mm.

During each experiment, the height of the layer in channel 1 was maintained constant, as was the pressure in vessel 3 below the hole. The material flowed from the channel through the discharge hole and accumulated in the vessel. During or after the experiment, the material was removed through hole 5 to the collector 6. The weight flow rate was determined by weighing the material that flowed through the discharge hole within a measured time interval. Equation (10) was used to determine the coefficient k_p; the gravitational flow rate was provided for on the basis of (7). The pressure differential was taken to be positive if the gas and the loose material flowed in the same direction, and negative if they flowed in opposite directions.

It was established that under the above conditions, k_p depends on the ratio of the areas of the channel and discharge hole, and can be found as

$$k_p = 0.78 + 0.22\,(D_{gr}/D)^2. \tag{11}$$

Figure 2 shows that the experimental data obtained for D/d = 7 to 200, D_{vc}/D = 1 to 11.5, H = 100 to 500 mm, and ΔP = −4000 to +4000 N/m^2 are generalized by Eq. (10), where k_p is found from (11), and G_{gr}

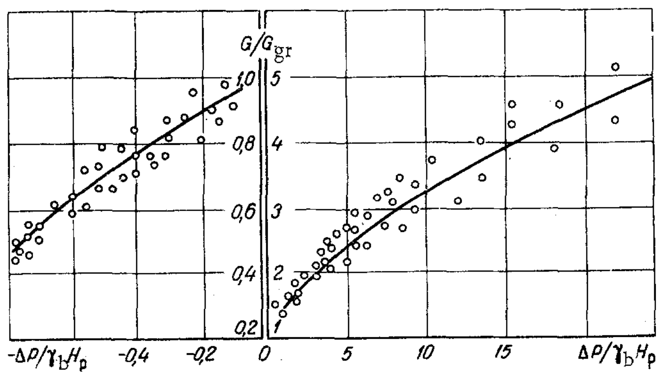

Fig. 2. Generalization of experimental data.

TABLE 3. Comparison of (12) with Experimental Data Obtained in [6] for D_{VC} = 203 mm.

D, mm	ΔP, kg/m²	G, g/sec		Error, %
		[6]	Eq. (2)	
Sand, d=0. 281 mm; γ_b = 1370 kg/m³				
6,35	3515	134	118	—12,0
	4218	138	127	— 6,5
	7733	200	172	—14,0
Saran, d = 0. 22 mm γ_b = 640 kg/m³				
6,35	7030	108	120	+11,0
	7030	113	120	+ 6,2
	9120	130	137	+ 5,4
12,7	4218	282	350	+25,5
	6000	350	410	+17,2

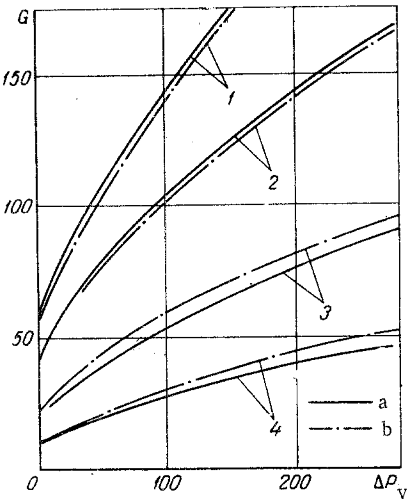

Fig. 3. Data from a) Eq. (2); b) [7] (G, g/sec; ΔP_V, kg/m²): 1) sand, d = 0.456 mm; γ_b = 1590 kg/m³; D = 14.1 mm; 2) KCl, d = 0.67 mm; γ_b = 1230 kg/m³; D = 14.1 mm; 3) sand, d = 0.68 mm; γ_b = 1590 kg/m³; D = 10 mm; 4) sand, d = 0.456 mm; γ_b = 1590 kg/m³; D = 7.1 mm.

from Eq. (7); the average error is ±8%, and the greatest error ±20%. In Fig. 2, we let H_p represent $\{[(H-D)/k_p] + D\}$.

The resulting relationship

$$G = \frac{2.15\gamma_b D^{2.5}}{1+11.8(d/D)} \sqrt{1+\frac{\Delta P}{\gamma_b\left[\frac{H-D}{0.78+0.22(D_{gr}/D)^2}+D\right]}} \qquad (12)$$

is suitable for determining the loose-material flow rate in the presence of a counter-directed gas flow (ΔP taken with minus sign), with gas flowing in the same direction (ΔP taken with plus sign), and for free gravitational flow (ΔP = 0). When Eq. (12) is used for flow in opposite directions, we must remember that when $-\Delta P/\gamma_b H_p \to -1$, the experimental data have poor reproducibility, so that this equation can be used for $-\Delta P/\gamma_b H_p > -0.8$.

Let us compare our experimental data with those of similar studies.

In [6], values of the mean weight flow rate are given over a time interval during which the height of the layer was not held constant; the initial spilling level was known, but the level at the end of each experiment was not indicated. It can be assumed, however, that for high rates of flow, where all of the material left the vessel within 30-100 sec, the final layer height was small, amounting to 50-100 mm. In such case, we can use Eq. (12) to compute the mean weight flow rate.

Table 3 shows values computed from (12), together with the experimental data of [6]; as we see, the error in the predicted values ranges from −14 to +25.5%.

Certain special equations have been proposed in [7] for determining the rate at which loose material flows through a hole in the horizontal bottom of a vessel for which D_{VC} = 140 mm, with excess pressure above the layer. Each of these equations is suitable for one particular material and one particular hole diameter; the pressure differential "near the hole" is used as the determining quantity.

It was established that the values found in this study for the pressure differential "near the hole" differ by no more than 30% from the values of ΔP_V computed from (3) with the aid of (4) and (11).

Thus the pressure differential "near the hole" and ΔP_V were taken to be equal. On this basis, in Fig. 3 we have compared the curves representing the equations of [7] and Eq. (2), which was used in the derivation of (12), and in which $\mu_p = \mu(1-\varepsilon_V)/(1-\varepsilon_{st})^{0.5}$ was found from Eq. (9); it was assumed that $(1-\varepsilon_V)/(1-\varepsilon_{st})h_V = D$.

As we see from Fig. 3, the discrepancy between the curves does not exceed 20%.

Finally, a graph has been given in [10] for refining the flow rate of a cracking-process catalyst under counter pressure and for $D_{vc} = D$; it corresponds, to within 12%, to the left side of the graph shown in Fig. 2 for Eq. (10), if we assume that $H = H_p$ for $D_{vc} = D$.

Thus Eq. (12), although relatively uncomplicated, is quite universal and is satisfactorily accurate, as is indicated by our experimental data and those of [6, 7, 10]; it can be used to determine the influence of air filtration on the flow of ideal free-flowing materials through a hole in a horizontal plane.

NOTATION

D is the hole diameter;
D_{vc} is the diameter of the vertical channel;
d is the theoretical particle size;
G is the weight flow rate of the loose material in unit time;
g is the gravitational acceleration;
F is the area of the hole;
H is the height of the layer in the vertical channel;
h_v is the height of the dynamic vault;
k is a coefficient;
ΔP is the overall pressure differential;
ΔP_v is the pressure differential between the upper and lower levels of the vault;
γ_v is the bulk density of the loose material;
γ is the specific gravity of the material;
ε_v is the average porosity in the space beneath the vault;
ε_{st} is the porosity of the stationary layer;
μ is the discharge coefficient.

Subscripts

gr is the gravitational flow;
p is the predicted value.

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